Mathematical modeling of individual parameters of the sum of the sizes intervertebral discs of the lumbar spine in juvenile males and males of the first mature age in norm

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Introduction

More than 200 million people are suffering from osteoporosis and musculoskeletal system diseases, according to WHO, occupy 4th place as a cause of disability and mortality in the world [13]. According to recent statistics from the International Osteoporosis Foundation, worldwide, 1 in 3 women over the age of 50 years and 1 in 5 men will experience osteoporotic fractures in their lifetime [25]. The experts predict an epidemic of osteoporosis, indicating the aging of the world population [5, 9]. Every fifth inhabitant of the globe suffers from back pain. In Ukraine chronic musculoskeletal system is also one of the most frequent problems [1, 15], and about 3.5 million people really familiar with the problems of the musculoskeletal system, it's terrible complications [11, 12], which require continuous long-term therapy [17, 18]. Degenerative disc diseases are an insidious illnesses that carefully hiding the other diseases symptoms, which significantly complicates their diagnosis, especially in the early stages.

The objective linear anterior-posterior diameter (APD), frontal diameter (FD) and vertical size (VS) of intervertebral discs, are not used in medical practice as criteria for evaluating their deviation from norm. Existing normative standards of intervertebral discs (IVD) linear sizes can be find in particular scientific works only and are not widely...
used in medical practice. Thus, a method for evaluating the partial dimensions of the lumbar IVD [6] submits a calculation of the average values of the IVD sizes without taking into account their gender differences and individual anthropometric measures. Setting of relative measures - the only way to take into account the personal characteristics of each human body, which makes it possible to estimate norms or the early-onset pathological alterations [7, 23].

Mathematical models as relative indicators for modeling of the personal norm as a whole exist [8, 22, 26]. However, they ensure for the estimation of sagittal and transverse size only and don’t include IVD height index, which is a key of pathological changes expression.

Meanwhile, the multiple supplementary anthropometric dimensions are difficulty identified and applied, that significantly restricts their widespread clinical use.

The purpose of the study: based on mathematical modeling to explore the individual parameters of the total size of the lumbar IVDs in juniors and men of the first adulthood (17-28 years) in norm.

Materials and methods
The 74 juniors and men of the first adulthood (17-28 years) without clinical manifestations of spine diseases were included in observation team. Individual comprehensive anthropometric investigation was performed for each research teams’ persons and measured general (body length and body weight) and particular sizes (the longitudinal, the transverse and the circumference sizes, and skin-fat folds’ thickness); lumbar spine MRI with the measuring of anterior-posterior diameter, frontal diameter and vertical size of the lumbar IVD was performed. Mathematical modelling by determining of the relative proportional nonlinear somato-disc value (based on body length and body mass) and calculating the total size of lumbar IVDs for every single person included in the study was undertaken. The TS of lumbar IVDs, which calculated as a sums of the anterior-posterior diameters, frontal diameters and vertical sizes of the lumbar IVDs and math modeling obtained dates were compared.

Mathematical processing of the measured parameters and the relative values of the somato-disc relationships was carried out by the statistical data processing program "STATISTICA 6.1" using parametric methods. The correct distribution of the variational series indicators, mean values and their standard errors were evaluated. The stepwise regression analysis method was used to create a mathematical model and predict the individual dimensions of the lumbar IVDs in practically healthy juniors and first adulthood males, depending on the body mass-growth parameters.

Results
The correlating of the external body measurements and the parameters of IVDs gives an affirmative answer to the question about their quantitative rates proportionality. Nevertheless, the correlations are too common and straight up, and unconnected with polymorphic structures. A clearer definition of multiple correlations (covariates) is a regression analysis that makes it impossible to determine the covariates of vertebral dimensional data with somatotypes. Regression analysis more strongly and accurate determines the proportional interrelations between body parameters and parameters of IVD.

Creation of the most optimal regression polynomial in predictability, a number of indicators, and the opportunity of logic verbal explanation is the major challenge of the investigation. The dependent variables are the lumbar IVDs measures (the IVDs total sizes as a sum of linear sizes of the APD, FD and VS of the lumbar IVDs). Graphical presentations depict interrelations between the IVDs total sizes of L1-L2 lumbar segments and the weight-length rates with a 95% confidence interval and show possibility to predict data of 72 from 74 cases in the given limits of reliability (Fig. 1).

The nature of the location is systemic and indicate the availability of patterns. Scattering diagram of the IVDs total sizes of the L1-L2, L2-L3, L3-L4 lumbar segments and relative magnitudes of the related mass-growth rate at a 95% confidence interval, they are possible to predict 72 cases out of 74 (Fig. 2).

Actually, the interrelation is at a higher level for relative terms - the relative magnitude of the weight-length rate and the IVD total size L1-L2 segment (IVD1) from the weight-length rate (Fig. 3).

Graphical presentations depict interrelations between the IVDs total sizes of L1-L2, L2-L3, L3-L4 lumbar segments and the weight-length rates with a 95% confidence interval and show possibility to predict data of 73 from 74 cases in the given limits of reliability (Fig. 4).

The direct stepwise regression analysis requires particularly fulfilling several conditions. Firstly, is the value of the F-criterion, which must be at least 2.00, and the final regression polynoms result should have a coefficient of determination (R²) at least 0.80, it means predictability more
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for the all lumbar IVDs of the L₁-L₅ segments. Determining the relative proportional nonlinear somato-disc values and the weight-length rates for each person in norm were carried out. The regression equations composition and characteristics for each listed lumbar IVD are summarized in Tables 1-4.

Coefficient of determination $R^2$, as a quality indicator of proximity (degree of predictability) is above 80.4%, which allows predicting dependent variable with the same rate. The standard error of the regression represents the average distance of observed values scattering fall from the regression line, in this case is 0.165.

According to Table 1, the final regression model is as follows:

$$K/SD_1 = (6.705+0.051*m-3.520*H)$$

(predictability is 80.4%), where (here and thereafter): $SD_1$ is the total size of IVD₁ in norm (MRI data) in cm; $K$ - weight-length rate (in kg/m); $m$ - body weight (in kg); $H$ - body length (in meters).

According to Table 2, the final regression model is as follows:

$$K/SD_2 = (6.184+0.049*m-3.243*H)$$

(predictability is 84.6%).

According to Table 3, the final regression model is as follows:

$$K/SD_3 = (6.592+0.048*m-3.475*H)$$

(predictability is 87.3%).

According to Table 4, the final regression model is as follows:

$$K/SD_4 = (6.822+0.047*m-3.612*H)$$

(predictability is 86.02%).

F-statistic is a global test to check for model with critical value 2.42 for number of variations, which in our case, F is from 91.27 - 152.35 (that is much more than the critical value, on the basis of which we can assert that the regression linear polynomial is significant ($p <0.000001$). The deviations and predicted values graph demonstrates the almost linear dependence, so the suggestion of a normal distribution of errors is complete, that is, the model...
is an appropriate (Fig. 5, 6).

Check the adequacy of regression models was defined in the analysis of differences between the predictive and relative actual parameter values to each other. Obviously maximum relative deviations make up generally no more than 9%.

Fig. 4. Scattering diagram of the weight-length rates relative magnitudes and the IVD total size L2-L5 segments (D2, D3, D4) from the weight-length rate.

is an appropriate (Fig. 5, 6).

Table 1. The final results of direct stepwise regression analysis of the relative weight-length rate and the IVD total size L1-L2 segment using the body length and weight juniors and men of the first adulthood (17-28 years) in norm.

<table>
<thead>
<tr>
<th>N=74</th>
<th>BETA Standard error BETA</th>
<th>B Standard error B</th>
<th>t(71)</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free index</td>
<td>6.705</td>
<td>0.831</td>
<td>8.064</td>
<td>0.0000</td>
</tr>
<tr>
<td>Body length (in m)</td>
<td>-0.513</td>
<td>0.075</td>
<td>-3.520</td>
<td>0.514</td>
</tr>
<tr>
<td>Body weight (in kg)</td>
<td>1.011</td>
<td>0.075</td>
<td>0.051</td>
<td>0.003</td>
</tr>
</tbody>
</table>

Table 2. The final results of direct stepwise regression analysis of the relative weight-length rate and the IVD total size L2-L3 segment using the body length and weight juniors and men of the first adulthood (17-28 years) in norm.

<table>
<thead>
<tr>
<th>N=74</th>
<th>BETA Standard error BETA</th>
<th>B Standard error B</th>
<th>t(71)</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free index</td>
<td>6.184</td>
<td>0.693</td>
<td>8.926</td>
<td>0.0000</td>
</tr>
<tr>
<td>Body length (in m)</td>
<td>-0.503</td>
<td>0.066</td>
<td>-3.243</td>
<td>0.428</td>
</tr>
<tr>
<td>Body weight (in kg)</td>
<td>1.038</td>
<td>0.066</td>
<td>0.049</td>
<td>0.003</td>
</tr>
</tbody>
</table>

Table 3. The final results of direct stepwise regression analysis of the relative weight-length rate and the IVD total size L3-L4 segment using the body length and weight juniors and men of the first adulthood (17-28 years) in norm.

<table>
<thead>
<tr>
<th>N=74</th>
<th>BETA Standard error BETA</th>
<th>B Standard error B</th>
<th>t(71)</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free index</td>
<td>6.592</td>
<td>0.607</td>
<td>10.84</td>
<td>0.0000</td>
</tr>
<tr>
<td>Body length (in m)</td>
<td>-0.558</td>
<td>0.060</td>
<td>-3.475</td>
<td>0.376</td>
</tr>
<tr>
<td>Body weight (in kg)</td>
<td>1.049</td>
<td>0.060</td>
<td>0.048</td>
<td>0.003</td>
</tr>
</tbody>
</table>

Table 4. The final results of direct stepwise regression analysis of the relative weight-length rate and the IVD total size L4-L5 segment using the body length and weight juniors and men of the first adulthood (17-28 years) in norm.

<table>
<thead>
<tr>
<th>N=74</th>
<th>BETA Standard error BETA</th>
<th>B Standard error B</th>
<th>t(71)</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free index</td>
<td>6.822</td>
<td>0.627</td>
<td>10.87</td>
<td>0.0000</td>
</tr>
<tr>
<td>Body length (in m)</td>
<td>-0.589</td>
<td>0.063</td>
<td>-3.612</td>
<td>0.388</td>
</tr>
<tr>
<td>Body weight (in kg)</td>
<td>1.038</td>
<td>0.063</td>
<td>0.047</td>
<td>0.003</td>
</tr>
</tbody>
</table>
As the second step of modeling, the further algebraic converting of the obtained proportions equation to determine the TS of three linear sizes for each lumbar IVDs in norm from the values of body weight and length. Accordingly, after transferring the total sum of linear dimensions to the left-hand side of the equation, we obtained the following final results:

\[
\begin{align*}
SD_1 &= \frac{K}{6.705 + 0.051m - 3.520H} \pm 9\% \\
SD_2 &= \frac{K}{6.184 + 0.049m - 3.243H} \pm 9\% \\
SD_3 &= \frac{K}{6.592 + 0.048m - 3.475H} \pm 9\% \\
SD_4 &= \frac{K}{6.822 + 0.047m - 3.612H} \pm 9\%
\end{align*}
\]

where: 
- SD - the sum of linear sizes IVD1 in norm (MRI measurement) in cm; 
- SD - sum of linear sizes of IVD1 2 in norm (MRI measurement) in cm; 
- SD - sum of linear sizes of IVD1 3 in norm (MRI measurement) in cm; 
- SD - sum of linear sizes of IVD1 4 in norm (MRI measurement) in cm; 
- K - weight-growth rate (in kg / m); 
- m - body weight (in kg); 
- H - body growth (in meters).

The modeling is carried out by determining the relative proportional somato-disc rates based on the weight-growth rate and the calculation of the TS of the three linear sizes of personal IVDs of the L₁-L₅ segments in norm for each specific individual.

The standard error for these models is ±5.0%. The established math models based on stepwise regression analysis and algebraic proportions transformation allow us to determine the sum of standard MRIs of the sizes IVD₁, IVD₂, IVD₃ and IVD₄ of L₁-L₅ segments in the norm on the basis of available in practical medicine anthropometric method - determination of mass and body length from further obtaining a relative weight-length rate.

Non-linearity dependence of the amount of the three standard sizes IVD₁, IVD₂, IVD₃ and IVD₄ in norm and the weight-length rate is presented in the graphs (Fig. 7, 8).

Most predicted TS lumbar IVDs and the sum of measures obtained by MRI-morphometric examination does not significant difference and is within the 9% limit. The quantitative values of the deviations are presented in the diagram (Fig. 9).

The common trend of calculated personal measures is to reduce the range of linear studied parameters and
can reasonably be said, that individual normal ranges are within ±9% of the calculated ones.

**Discussion**

The use of the proposed approach allows direct prognostic evaluation of the total value of standard linear MRI dimensions of the lumbar IVDs to diagnose early stages of intervertebral disc disease by MRI and CT scans.

There are examples in the scientific literature of the use of modeling to calculate the individual parameters of intervertebral discs and vertebral bodies [2, 3, 10, 19, 21, 26].

The interrelation of such individual factors as gender and age with linear dimensions of the spine and its' individual elements has been described, but few studies have considered the importance of other individual factors: body length, body weight and other anthropometric dates [16].

Some researchers noted the low correlation between anthropometric data and the cross-sectional area (CSA) of the vertebral endplates, and therefore expressed the inability to predict the area of the locking plates by anthropometric parameters; however, clear correlations were found in the size of the intervertebral discs and the

**Fig. 7.** Non-linear correlation of the TS of lumbar IVD1 and weight-length rate juniors and men of the first adulthood (17-28 years) in norm.

**Fig. 8.** Non-linear correlations of the TS lumbar IVD2, IVD3, IVD4 and weight-length rate juniors and men of the first adulthood (17-28 years) in norm.

**Fig. 9.** Quantitative deviations values of real TS lumbar IVDs measured by MRI juniors and men of the first adulthood (17-28 years) in norm from the predicted values.
vertebral endplates [20].

D. Colombini et al. [4] found that the bone component of the body mass and wrist diameter had better correlation with the CSA of the lower lumbar intervertebral discs compared to the body weight index.

Z. Turk and D. Celan [27] proposed the mean square value of the width of the wrist, elbow and knee joint structures with the exception of the ankle diameter at the bone level to modeling the value of the CSA vertebrae endplates.

Degeneration of the intervertebral disc is a time-consuming process with a tendency to progress, so it is important to diagnose the earliest disc degeneration [24], including with early changes of the vertebrae endplates [14].

The automated systems programme development for determining of the linear organs’ parameters, among them intervertebral discs, with minimal staff involvement in data processing, is a modern, promising method for automated evaluation of medical images [3, 28].

The analysis of the diagnostic value of the lumbar IVD total sizes modeling is showed high efficiency for detection of chondrosis and degenerative IVD changes.

The relevance and sensibility of the finding formulas in patients with various forms of acquired lumbar IVDs diseases which no abnormalities were detected in terms of linear parameters of intervertebral discs, calculated by existing methods. The assurance team was included juniors and men of the first mature age (17-28 years) with lumbar back pain, which had previously been diagnosed with chondrosis and disc degeneration. MRI of the lumbar spine were performed for respondents and measured the vertical, sagittal and transverse dimensions of the lumbar IVD between the bodies $L_1$, $L_2$, $L_3$, $L_4$, $L_5$, $L_6$. The personal parameters of the TS of the lumbar intervertebral discs for each respondent was modeled by the weight and growth data. Clinical testing of the model demonstrated significant difference TS IVDs in 46% of patients with a diagnosis of chondrosis and degenerative disc disease and the individually calculated TS IVDs. The obtained data represents the high sensitivity and informativity of the math modeling method.

Mathematical modeling, as a promising direction, can serve as a basis for calculating as for the individual standard linear sizes of the lumbar intervertebral discs, as the individual normal somato-organ parameters in the norm, which may serve as additional criteria for the detection of pathologies in the early stages, including automated diagnostic imaging systems.

Conclusions

1. Mathematical modeling can be basis for calculation of the individual normal parameters of the total size of the lumbar intervertebral discs (IVD) in juniors and men of the first mature age (17-28 years).

2. At the core of the model analysis lies the total size of the lumbar IVD three dimensions in a normal person, founded on the value of the weight-growth rate, which includes the of the previously created regression model of the relative proportional coefficient of the weight-growth rate per unit intervertebral discs' total sizes between 1 to 5 lumbar vertebrae. Modeling is performed by calculating of the relative proportional nonlinear somato-disc value founded on the weight-growth rate for each person in norm.

3. The investigation of standard linear measures of the lumbar IVD allows to diagnose the early stages diseases of the intervertebral discs by MRI and CT scans.

References


может помочь математическое моделирование, которое следует использовать для расчета индивидуальных линейных параметров внутренних структур на основе внешних параметров тела. Целью исследования было рассчитать индивидуальные суммарные линейные размеры межпозвонковых дисков поясничного отдела позвоночника у юношей и мужчин первого зрелого возраста (17-28 лет) в норме. Суммарный размер межпозвонковых дисков поясничного отдела позвоночника вычисляли как сумму измеренных межпозвонковых дисков поясничного отдела позвоночника: сагittalного размера, поперечного размера и вертикального размера, которые измеряли на магнитно-резонансных томограммах. Следующим шагом было вычисление относительных пропорциональных непропорциональных сомато-дисковых соотношений (основываясь на показателях массы и длины тела) для каждого исследуемого индивидуально. Математическую обработку измеренных параметров и относительных величин сомато-дисковых соотношений проводили программой статистической обработки данных "STATISTICA 6.1" с помощью параметрических методов. Оценивали правильность распределения показателей вариационного ряда, средние значения и их стандартные ошибки. На основе относительных показателей была создана математическая модель для получения индивидуальных величин суммарного размера межпозвонковых дисков поясничного отдела позвоночника. В дальнейшем провели сравнение измеренных суммарных показателей сагittalного, поперечного и вертикального размеров поясничных позвонков с математически рассчитанной величиной для каждого межпозвонкового диска поясничного отдела позвоночника. Достоверная разница математически рассчитанных и измеренных величин суммарных размеров межпозвонковых дисков не превышала 10%. Определение стандартных линейных размеров межпозвонковых дисков поясничного отдела позвоночника с помощью КТ и МРТ и сравнение с теоретически рассчитанными показателями позволяет диагностировать ранние проявления патологии межпозвонковых дисков поясничного отдела позвоночного столба.

Ключевые слова: моделирование, межпозвонковый диск, поясничный отдел позвоночника, норма, юноши.